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TECHNICAL NOTE

Time-varying-response smoothing

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Abstract

Cubic smoothing splines with a fixed-period response are used widely in producing "expected" growth curves for ring-width and density data in dendroclimatology. A simple modification to the procedure which generates these splines enables the use of a smoothing spline with a user-specified, time-varying flexibility and hence time-varying-response characteristics. The revised procedure is presented here, along with different examples of its application in the context of Regional Curve Standardisation (RCS). The ability to generate a smoothing spline with time-dependent flexibility may have wider application in tree-ring studies.
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Cubic smoothing splines

Data-adaptive smoothing of tree-ring series, or climate estimates based on them, is common practice in dendroclimatology. One example is in the specification of expected tree-ring-width (TRW) or maximum-latewood density (MXD) standardisation curves, where the most commonly used techniques involve the use of Gaussian filters or cubic smoothing splines (Briffa et al., 1987; Cook et al., 1990). The use of cubic smoothing splines to standardise series of tree-ring measurements was introduced to dendroclimatology by Cook and Peters (1981) and a procedure to fit a cubic smoothing spline, with a fixed-frequency cut off is incorporated within widely used dendroclimatic software, e.g. AR-STAN (Cook, 1985) and COFECHA (Holmes, 1983),

freely available from the Laboratory of Tree-Ring Research, University of Arizona. The smoothing spline is a series of piecewise cubic polynomials with a knot at each data point abscissa (Wold, 1974). The Lagrange multiplier limits flexibility to that specified by the user as a parameter, the 50% amplitude cut-off period (Peters and Cook, 1981). The spline procedure generates a series of simultaneous equations which systematically relate each year's measured value to the preceding year and following year. The sets of simultaneous equations are formed into a tri-diagonal matrix which is solved simultaneously. In its normal application in available tree-ring software, the flexibility of the smoothing spline is set to produce a fixed frequency response (Cook and Peters, 1981). However, because the flexibility of the fitted spline at each year is virtually independent of that for rings beyond its neighbours, it is possible to apply gradual variation in the effective frequency response through time and so specify a time-varying response. One possible application is described below.

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Producing RCS curves

The RCS method involves the production of a general curve that represents the expected magnitude of ring width (or MXD) for a given ring age of tree growing in a specified region, and is usually calculated for a particular species by averaging the measured values (representing a wide range of calendar years) after alignment by cambial age (Briffa et al., 1992). This empirically derived curve will invariably display some degree of high-frequency variance (in this case noise) resulting from inter-sample variability and is usually smoothed to create the applicable RCS curve (Briffa et al., 1992). Where tree counts are large (say several hundred as in Fig. 1a) the magnitude of remaining noise is negligible (though the uncertainty about the RCS curve may still be large) but where tree counts are relatively low (Figs. 1b and 1c) the magnitude of this noise can be important as it may inappropriately influence the definition of the RCS curve.

Provided correct allowance is made for pith offset in aligning the records by cambial age, the mean ring-width curve generally rises in the first decade or so of tree life and then decays steadily with increasing ring age (Fig. 1). The section of the mean-ring width curve corresponding to the oldest rings, where tree counts are often small, will have larger amplitude variance than the earlier sections. Smoothing these curves therefore, requires a method that is sufficiently flexible to follow the rapidly changing values of the initial rise in mean ring width and yet is sufficiently stiff so as not to be sensitive to the high-frequency variance in later years. A number of methods have been used previously. Examples include least squares-fitted modified negative exponential curves (Briffa et al., 1996; Gunnarson and Linderholm, 2002; Naurzbaev et al., 2002; Helama et al., 2005), the generalised negative exponential (i.e. Hugershoff) curve (Grudd et al., 2002), a cubic smoothing spline with flexibility of 10% RCS curve length (Esper et al., 2003; Büntgen et al., 2005), and for MXD sloping straight lines (Bräker, 1981; Briffa et al., 1996). These curves are generally either too stiff in the early (i.e. younger cambial age) years (e.g., Fig. 1b and 1c) or too flexible in later (older) years (e.g., Fig. 1a) where few
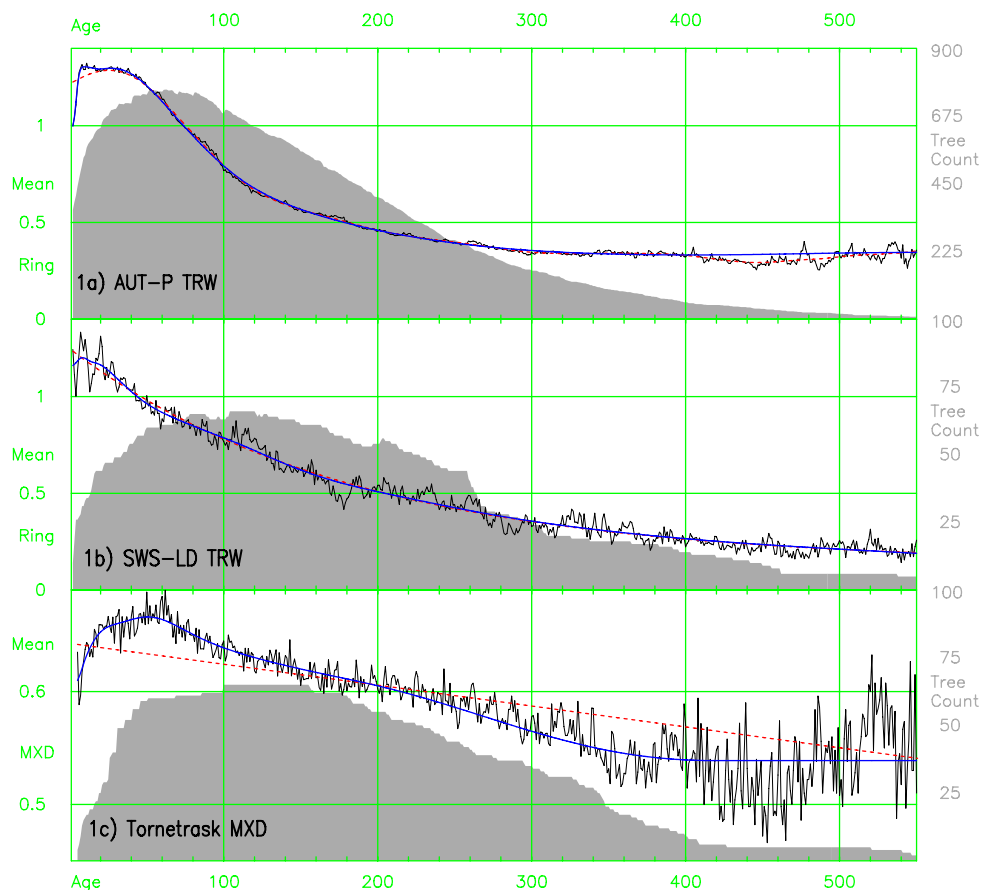

Fig. 1. Examples of mean measurements by ring age (black), smoothed regional growth curves created using age-dependent smoothing (blue) (see online version), and alternative smoothing curves (dashed red) of 1(a) cubic smoothing spline with flexibility of 10% RCS curve length (Esper et al., 2003), 1(b) fitted modified negative exponential curve (Fritts et al., 1969), and 1(c) a sloping straight line (Briffa et al., 1992).

cores are available, suggesting a requirement for variable-flexibility when smoothing the RCS curve. Observation of mean ring-measurement curves by age (Fig. 1) indicates that the 50% frequency dependent cut-off needs to be less than a decade or so for young RCS rings and several centuries for older sections. Here we suggest that this can be achieved using a minor adjustment in the application of the spline smoothing routine.

Age-dependent smoothing in RCS standardisation

The fixed-frequency response spline procedure was taken from the widely used standardisation program ARSTAN and modified (in FORTRAN 90) to provide a variable-frequency response. The user must specify values of the 50% frequency dependent cut-off para-meter (SSA) for each year and this may be constant as in existing applications or it can have annually varying values, for example for age-dependent smoothing SSA is set to ring age plus a constant, the initial stiffness. When age-dependent smoothing is applied to the mean cambial-age data, the resulting spline is flexible in the early years and becomes progressively stiffer in later years to satisfactorily retain the initial growth rise while having less sensitivity to the increased local variability in the later years. RCS curves based on TRW or MXD are not expected to increase in later years and an additional constraint, that of replacing any values after a "mini-mum" in the last third of the smoothed curve (if one exists) with the value of that minimum, is applied to prevent an increase in value at the end of the RCS curve. In the MXD example (Fig. 1c) there is an apparent increase in the last 50 years of the oldest three trees and the RCS curve is continued as a horizontal line from the minimum value. It is necessary to examine the mean curves by age and associated smooth curves to obtain a suitable initial stiffness for each RCS curve and ensure a satisfactory fit.

Examples

Fig. 1 illustrates 3 examples where mean ring-measurement curves, plotted against ring age, are smoothed to produce RCS curves. Fig. 1a is based on data from the last 2000 years (180 BC to AD 2002) of a multi-millennium ring-width chronology of living, his-torical and sub-fossil stone-pine trees (*Pinus cembra*) from high elevations in the Eastern Alps, AUT-P TRW

(Nicolussi and Schießling, 2001; Nicolussi et al., 2004). With several hundred trees, the 50% frequency cut off parameter was set to "2-years plus ring age" for this example (smooth blue curve). A cubic smoothing spline (red dashed curve) with a fixed-period response of length 10% that of the RCS curve length fits well between ring ages 10 and 400 but does not follow the early rise in ring-width and is distorted as it follows the medium-frequency variance in the final century. Fig. 1b uses ring-width data, covering the period AD 663 to 1998, comprising living and sub-fossil larch (*Larix decidua*) from high elevations in the Austrian Alps, SWS-LD TRW (Grabner et al., 2001, 2006). With fewer trees in this example, the 50% period cut-off parameter was set to 15 years plus ring age (smooth blue curve). A modified negative exponential curve (red dashed curve) fits well after ring age 20 but is a poor fit in the first two decades. Fig. 1c is based on MXD data, covering the period AD 441 to 1980, from a mixture of living and sub-fossil pine trees (*Pinus sylvestris*) from north Sweden, Tornetrask MXD (Schweingruber et al., 1988). Again with fewer trees, the 50% period cut off parameter was set to 15 years plus ring age (smooth blue curve). A sloping straight line (red dashed curve) is a poor fit to this mean MXD curve.

In Examples 1a and 1b the alternative curves are selected as the best of those available and age-dependent smoothing, although a better fit, will have little net effect on the resultant chronology. In Example 1c the alternative straight line fit was suitable when pith estimates were not used (Briffa et al., 1992) but here, with the use of pith estimates, the more flexible age-dependent smoothing provides a closer fit. The choice of smoothing should be based on how well the chosen curve fits the mean ring width curve. In this note we have provided one illustration of the potential use of varying response filters in dendroclimatology. Addi-tional procedures might be produced to use arrays of "varying stiffness" values designed to fit specific requirements in other applications, for example stiffness based on tree counts.

Acknowledgements

Appendix A. FORTRAN 90 procedure to fit a variable stiffness spline to annual data

```fortran
SUBROUTINE spline(n,rw,ssy,rwp)                                                                    ! Spline calculation
! ***************************************************************
! DERIVED FROM IMSL LIBRARY ROUTINES BY EDWARD R COOK,
! LAMONT-DOHERTY GEOLOGICAL OBSERVATORY, AND MODIFIED
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! ***************************************************************
IMPLICIT NONE
INTEGER,INTENT(IN)                              :: n                          ! Length of series
INTEGER,DIMENSION(n),INTENT(IN)                 :: ssy                        ! Spline stiffness each year
REAL,DIMENSION(n),INTENT(IN)                    :: rw                         ! Series to fit
REAL,DIMENSION(n),INTENT(OUT)                   :: rwp                        ! Spline curve
INTEGER                                         :: i
REAL(8),PARAMETER                               :: pi = 3.1415926535897935_8
REAL(8)                                         :: arg
REAL(8),DIMENSION(−1:n)                         :: a1,a2,a3,a4,p              ! Working arrays
DO i = 1,n−2                                                                  ! Stiffness for each year
arg = DCOS(2.0_8*pi/DBLE(ssy(i)))
p(i) = (2.0_8*(arg−1.0_8)**2)/(arg+2.0_8)
ENDDO                                                                        ! Set matrix values
a1 = 0.0_8 ; a2 = 0.0_8 ; a3 = 0.0_8 ; a4 = 0.0_8
a1(3:n−2) = 1.0_8 ; a2(2:n−2) = −4.0_8+p(2:n−2)
a3(1:n−2) = 6.0_8+p(1:n−2)*4.0_8
a4(1:n−2) = DBLE(rw(1:n−2)+rw(3:n)−2.0*rw(2:n−1))
DO i = 1, n−2                                                                ! Eliminate values
a1(i) = a1(i)*a3(i−2) ; a2(i) = (a2(i)−a1(i)*a2(i−1))*a3(i−1)
a3(i) = 1.0_8/DSQRT(a3(i)−a1(i)**2−a2(i)**2)
a4(i) = (a4(i)−a4(i−2)*a1(i)−a4(i−1)*a2(i))*a3(i)
ENDDO
DO i = n−2,1,−1                                                              ! Back substitute
a4(i) = (a4(i)−a4(i+1)*a2(i+1)−a4(i+2)*a1(i+2))*a3(i)
ENDDO                                                                        ! Build smooth curve
rwp(1:n) = rw(1:n)−SNGL(a4(−1:n−2)−2.0_8*a4(0:n−1)+a4(1:n))
RETURN
END SUBROUTINE
```

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