SM6: Chronology confidence
EC1. General
EC2. Smoothed chronologies v smoothed trees (Figures EC3-EC6)
EC3. Expressed Population Signal (Figures EC7-EC13)
EC1. General

Figure EC1 Sample counts over time for various sub-sets of the Yamalia data; a) Pou_la original larch sub-fossil data from 1991, (b) the Pou_la original larch living data from 1991, (c) Polurula update of the larch sub-fossil data from 1999; (d) the combined (Pou_la and Polurula) Polar Urals larch data, (e) the Yamal larch data of Hantemirov and Shiyatov (2002), and (f) the Yamal data used in this paper. Sample counts of cores are shown in grey and counts of mean-trees are shown in cyan. In (d) and (e) sample counts of less than ten are shown in red. The quality of the chronology (EPS) is based on between tree correlations and having duplicate measurements from individual trees does not significantly improve chronology confidence. The replication of the Polar Urals sub-fossil data (a) is very poor in the medieval period (before 1200 CE) and the additional Polurula samples (c) were generated in an attempt to improve this situation. The replication of the combined Polar Urals series (d) is still poor prior to 1100 CE compared to the replication of the Yamal data shown in (e) and (f).
Figure EC2 This figure illustrates one approach to representing the time-dependent chronology uncertainty at discrete frequency bands for the various TRW and MXD chronologies. The measurement data sets were standardised using two-curve, signal-free RCS with tree-indices transformed to have a normal distribution. Separate data sets were used for Yamal TRW (yml-all.raw), Polar Urals TRW (Polar.raw), and Polar Urals MXD (Polarx.xsd). The Yamalia TRW data were created by averaging the separately processed tree-index series from Yamal TRW and Polar Urals TRW. Separate frequency-limited chronologies were created by filtering the series of tree-indices and averaging the filtered series (see main text Section 4.5). For each chronology, the standard error was calculated for each year (where sample counts >3) by scaling the standard deviation by the square root of the sample count. Where sample counts are > 3, two standard errors are plotted either side of the mean for Yamal TRW (blue), Polar Urals MXD (cyan), Polar Urals TRW (red), and the combined Yamalia (Yamal plus Polar Urals) TRW (green). Errors are shown separately for (a) all frequencies, (b) 15-year high-pass, (c) 15 to 100-year band pass, and (d) 100-year low pass chronologies.
**EC2. Smoothed chronologies v smoothed trees**

These Figures illustrate the different ways of producing smoothed chronologies at different frequencies. The measurement data sets were standardised with two-curve, signal-free RCS with tree-indices converted to have a normal distribution. Separate data sets were used for Yamal TRW (yml-all.raw), Polar Urals TRW (Polar.raw), and Polar Urals MXD (Polarx.mxd). The Yamalia TRW data were created by averaging the separately processed tree-index series from Yamal TRW and Polar Urals TRW. Chronologies were filtered to produce 15-year high-pass, 15 to 100-year band-pass, and 100-year low-pass chronologies. Another set of filtered chronologies was created by filtering the series of tree-indices and averaging the filtered series.

**Figure EC3** Comparison of chronologies directly smoothed (red) and chronologies developed by averaging smoothed series of tree indices (blue) for Yamal TRW: the full chronology (a) containing all frequencies, b) 15-year high-pass, c) 15 to 100-year band-pass, and d) 100-year low-pass.
Figure EC4 Comparison of chronologies directly smoothed (red) and chronologies developed by averaging smoothed series of tree indices (blue) for Polar Urals TRW. The full chronology is shown in (a) containing all frequencies, b) 15-year high-pass, c) 15 to 100-year band-pass, and d) 100-year low-pass.
Figure EC5 Comparison of chronologies directly smoothed (red) and chronologies developed by averaging smoothed series of tree indices (blue) for Polar Urals MXD. The full chronology is shown in (a) containing all frequencies, b) 15-year high-pass, c) 15 to 100-year band-pass, and d) 100-year low-pass.
**Figure EC6** Comparison of chronologies directly smoothed (red) and chronologies developed by averaging smoothed series of tree indices (blue) for Yamalia TRW. The full chronology is shown in (a) containing all frequencies, b) 15-year high-pass, c) 15 to 100-year band-pass, and d) 100-year low-pass.
EC3. Expressed Population Signal
Expressed Population Signal (EPS) is commonly used to assess the changing level of statistical confidence of a chronology. Standard methods of calculating EPS are frequency limited in the sense that they represent high-frequency chronology confidence accurately but provide little information about the reliability of the low-frequency components of chronology variance. Here we use a method designed to estimate EPS for chronologies created using RCS and hence containing a large proportion of long-timescale information (Melvin and Briffa 2013). A detailed description of the method used and results of applying this method to the Yamalia chronologies are shown below.

Estimate of EPS for RCS chronologies
The Expressed Population Signal (EPS) gives an indication of the proportion of chronology variance that represents chronology signal which is common signal: the variance shared in common among the constituent tree-index series. This is represented as a value between zero and 1.0 and is estimated as the mean correlation coefficient for the inter-comparison of all mean-tree index series (rbar). Rbar can be calculated in different ways on the basis of maximum overlap comparisons between all index series; or calculated from a fixed overlap period; or calculated for a running period of fixed length to acquire a better indication of transient signal strength. The remaining function of variance (1.0 – rbar) represents variance that is not common among the different tree-index series and is therefore considered a measure of statistical “noise”, which is reduced in the chronology in proportion to the number of sample index series averaged at any time (Wigley et al. 1984, Briffa and Jones 1990, Jones et al. 2009). EPS is calculated as:

$$EPS = \frac{rbar \times \text{Count}}{1.0 + (\text{Count} - 1.0) \times rbar}$$

Rbar can only be assessed between overlapping trees and consequently is frequency limited, to periods shorter than the period of overlap used in calculating rbar. In common practice, EPS is only relevant for assessing high-frequency chronology confidence, e.g. typically a 50-year running window might be used for rbar calculation.

An estimate of the additional uncertainty in an RCS chronology compared to a 50-year spline chronology can be obtained using the ratio of the standard deviations (SD) of the two chronologies. Here we estimate an effective count (Eff. Count) for use in calculating EPS for RCS chronologies, where:

$$\text{Eff. Count} = \frac{\text{Count}}{\left(\frac{\text{RCS SD}}{\text{Spline SD}}\right)^2}$$

Because the standard deviation of RCS chronologies generated as fractional deviations (especially in the case of TRW without the transformation of tree indices to have a normal distribution) is proportional to the chronology index value we use adjusted standard deviation (ASDev = Sdev / index) for chronologies derived from fractional deviations.

The Figures below show in top panel rbar calculated for a 50-year spline chronology (black) and an RCS chronology (red). The lower panel shows EPS calculated for a 50-year spline chronology (black), for an RCS chronology (red), and EPS for an RCS chronology which has been adjusted for the effective count and has scaled standard deviation (blue) as described above. The EPS calculated using effective count and adjusted standard deviation gives what we think is good estimate of EPS (the proportion of signal) for RCS chronologies.
**Figure EC7** – EPS calculation for Polar Urals TRW (Polar.raw) chronology created using two-curve, signal-free RCS showing EPS for 50-year spline (black) and EPS for RCS (blue).

**Figure EC8** – EPS calculation for Polar Urals MXD (Polarxs.mxd) chronology created using two-curve, signal-free RCS showing EPS for 50-year spline (black) and EPS for RCS (blue).
**Figure EC9** EPS calculation for Yamal TRW (yml-all.raw) chronology created using two-curve, signal-free RCS showing EPS for 50-year spline (black) and EPS for RCS (blue).

![Graph](image1)

**Figure EC10** EPS calculation for the Yamal TRW data set (Yamalad.raw) used by Briffa (2000) for the chronology created using two-curve, signal-free RCS showing EPS for 50-year spline (black) and EPS for RCS (blue).

![Graph](image2)
**Figure EC11** EPS calculation for Polar Urals TRW (Polar.raw) chronology created using two-curve, signal-free RCS with tree indices transformed to have a normal distribution showing EPS for 50-year spline (black) and EPS for RCS (blue).

**Figure EC12** EPS calculation for Polar Urals MXD (Polarxs.mxd) chronology created using two-curve, signal-free RCS with tree indices transformed to have a normal distribution showing EPS for 50-year spline (black) and EPS for RCS (blue).
Figure EC13 EPS calculation for Yamal TRW (yml-all.raw) chronology created using two-curve, signal-free RCS with tree indices transformed to have a normal distribution showing EPS for 50-year spline (black) and EPS for RCS (blue).